Astrophysical Censorship

Umpei Miyamoto,* **Sanjay Jhingan,†** **Tomohiro Harada***

**Department of Physics, Rikkyo University, Tokyo 171-8501, Japan*
†Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi 110025, India

umpei@rikkyo.ac.jp sanjay.jhingan@gmail.com harada@rikkyo.ac.jp

Abstract

There are several solutions to the Einstein equation exhibiting naked-singularity formation in gravitational collapse, which could possibly serve as counterexamples to the cosmic censor hypothesis. It has not been examined seriously, however, whether or not such naked singularities are indeed visible to asymptotic observers in astrophysically realistic stellar collapse, such as the core collapse and the delayed collapse in the final stage of its evolution. In this paper, we set the spherically symmetric mass distribution which is motivated by the astrophysical scenarios and evolve it with vanishing pressure for simplicity. We show that even if a naked singularity could appear at the centre, the naked singularity is hidden behind an event horizon and cannot be observed by a distant observer. With this illustration, we argue the existence of an "astrophysical censor" that prohibits singularities formed in astrophysical gravitational collapse from being observed by a distant observer.

Contents

1 Introduction **1**

2 Spherical dust collapse **3**
2.1 Metric, singularity, and apparent horizon 3
2.2 Initial conditions for stellar collapse 4
2.3 Null geodesic equation 5

3 Astrophysical censorship **6**
3.1 Weak censorship ($n = 1$ and $n = 2$) 7
3.2 Strong censorship ($n = 3$) 8

4 Conclusion **9**

1 Introduction

Singularity theorems have been one of the most important developments in the theory of classical general relativity [1]. These theorems ensure that under very general and physically reasonable conditions singularities are bound to occur in nature. Such singularities may occur in a cosmological model or gravitational collapse of a star. In physical terms a singularity is a point of spacetime where curvatures become so large that laws of classical physics do not hold. Mathematically it is not a point in the spacetime manifold. To ensure the validity of classical laws of physics, a hypothesis that a physically realistic gravitational collapse within classical general relativity will never result in a naked singularity, called cosmic censorship [2], was proposed. By a globally naked singularity we mean the existence of future-directed non-spacelike curves from a singularity to a far-away observer, implying the violation of weak censorship. If all the future-directed non-spacelike curves fall back into the singularity we only have the violation of strong censorship, where the singularity is termed as a locally naked singularity.

The strong censorship has been shown to be false in the interior of charged black holes in the Einstein-Maxwell-scalar system numerically [3] and rigorously [4]. Moreover, there has been a growing family of examples where naked singularities form in the spherically symmetric collapse of a dust cloud [5,6] and a perfect fluid with nonvanishing pressure [7], leaving a subtle problem whether these examples are "physically realistic". See [8,9] for other interesting examples. The naked singularities seem to arise not only in various matter models (dust, null dust, perfect fluids, scalar fields, Yang-Mills fields) but also under various symmetries (spherical, quasi-spherical, cylindrical, numerical studies of axially symmetric configurations). All these models exhibit the formation of naked singularities as well as black holes. The numerical studies of gravitational collapse have also unveiled intriguing phenomenon of critical behavior [10]. Therefore, the non-existence of a general solution to Einstein equations which led the studies on gravitational collapse to specific models has yielded rich dividends. In fact, with so many possible counterexamples we cannot take the cosmic censorship as a physical principle but only as a rather mathematical question of whether or not these naked-singular spacetimes are generic in the space of all possible initial data sets. Together with a general belief that general relativity should be replaced by quantum gravity in the Planckian regime, it has to be said that the original cosmic censorship is obsolete and should be accordingly modified (e.g., see [11,12]).

If the cosmic censorship is violated in any sense, it would be important whether the violation can be tested experimentally or observationally. The possibility of the cosmic censorship violation in the particle collider has been recently discussed [13]. Studies of the cosmic censorship hypothesis from an astrophysical point of view are becoming an active research subject. In particular, many researchers are interested in the relevance and consequence of an overspinning Kerr spacetime, which is a naked singularity in classical general relativity [14]. As for the possible formation of naked singularity in astrophysical situations, the result of numerical relativity

in its early development suggests that the cosmic censorship is preserved in the collapse of the evolved star with large angular momentum under plausible initial conditions. See [15] and references therein. Giacomazzo et al. [16] recently studied the fate of a rapidly differentially rotating neutron star with the angular momentum greater than the Kerr bound and found that generic conditions for such a progenitor do not lead to a naked singularity.

Naively, in the core collapse of a very massive star or the delayed collapse of a newly born neutron star, the behaviour of the collapse is understood as free fall once the dynamical instability sets in and the collapse proceeds. This suggests that we can approach the collapse problem by assuming a pressureless fluid or dust. For this reason, we hereafter will concentrate on the spherically symmetric dust model as a first step. The studies so far have mainly concentrated on the issue of local visibility and geodesic analysis near the singularity, although there have been a few studies analyzing global structures of naked singularity [5, 17]. This introduces a considerable simplification since the formation of naked singularity depends on the matter distribution only in the central region. In fact, one can easily construct both locally and globally naked singular models when the central matter distribution admits naked-singularity formation, if one is allowed to put the mass distribution in the surrounding region arbitrarily.

In this paper we analyze the formation of singularity and its causal structure based on the astrophysically realistic collapse scenarios in the final stage of stellar evolution, such as the core collapse and the delayed collapse. The idea is, with so many examples showing the formation of naked singularities, to find a suitable physical condition under which the censorship may be preserved and study whether or not the astrophysically realistic collapse satisfies this condition. One direction could be to impose additional physical conditions on the models showing possible counterexamples. However, the imposition of any new condition other than genericity and energy conditions can amount to the restrictions which may be hard to justify. One can say that naked singularities are harmless in a sense if they do not disturb an asymptotic observer. For the asymptotic observer the violation of strong censorship causes no harm. Therefore, as for the direct testability of the violation of the cosmic censorship, it is important whether or not the weak cosmic censorship conjecture is preserved or not. With this perspective we analyze here the well known Lemaître-Tolman-Bondi model with the initial conditions where naked singularities are known to appear as an outcome of the evolution.

We begin with reviewing the basic framework of the dust-collapse model. The next section deals with the main results of this paper. We work in the geometrized unit ($c = G = 1$), otherwise denoted.

2 Spherical dust collapse

The most general solution describing a spherically symmetric dust ball is given by Lemaître-Tolman-Bondi metric [18]. It is a natural generalization of Oppenheimer-Snyder model which describes collapse of a homogeneous dust cloud [19].

2.1 Metric, singularity, and apparent horizon

The energy-momentum tensor of a pressureless fluid is written as $T^{\mu\nu} = \rho u^\mu u^\nu$, with the energy density ρ and the normalized velocity field u^μ ($u^\mu u_\mu = -1$). Solving the Einstein equations with the spherically symmetric ansatz, the Lemaître-Tolman-Bondi (LTB) solution is obtained in comoving synchronous coordinates ($u^\mu = \delta^\mu_t$) as

$$ds^2 = -dt^2 + \frac{R'^2}{1+f(r)}dr^2 + R^2(t,r)d\Omega^2, \tag{2.1}$$

$$\dot{R}^2 = f(r) + \frac{F(r)}{R}, \tag{2.2}$$

$$\rho(t,r) = \frac{F'}{8\pi R^2 R'}, \tag{2.3}$$

where $X' = \partial_r X$, $\dot{X} = \partial_t X$, and $d\Omega^2$ is the line element of a unit two-sphere. $F(r)$ and $f(r)$ are integration constants, and fixed once the initial distributions of mass and velocity of the fluid are specified. Their physical interpretation can be seen from equation (2.2): the left-hand side is a kinetic energy; the second term on the right-hand side plays a role of potential energy; $f(r)$ is a total energy. Integrating equation (2.2), we obtain $R = R(t,r)$ in an implicit form

$$t - t_s(r) = -\frac{R^{3/2}}{\sqrt{F}} G\left(-\frac{fR}{F}\right), \tag{2.4}$$

where $t_s(r)$ is a constant of integration and

$$G(y) = \begin{cases} \dfrac{\text{Arcsin}\sqrt{y}}{y^{3/2}} - \dfrac{\sqrt{1-y}}{y}, & 0 < y \le 1 \\ \dfrac{2}{3}, & y = 0 \\ -\dfrac{\text{Arcsinh}\sqrt{-y}}{(-y)^{3/2}} - \dfrac{\sqrt{1-y}}{y}, & -\infty \le y < 0 \end{cases} . \tag{2.5}$$

With the coordinate degrees of freedom available, one can choose initial time at $t = 0$ and fix $R(0,r) = r$ without any loss of generality. Then, the expression for time when a fluid element defined by a particular $r = const.$ value plunges into the shell-focusing singularity, which is defined by $R = 0$, is given by

$$t_s(r) = \frac{r^{3/2}}{\sqrt{F}} G\left(-\frac{fr}{F}\right). \tag{2.6}$$

For spherically symmetric systems the condition for the existence of an apparent horizon (i.e., an inner boundary of the region containing trapped surfaces), corresponds to the existence of two-spheres whose outward normals are null, is given by

$$\nabla_\mu R \, \nabla^\mu R = 0 \quad \Rightarrow \quad R = F. \tag{2.7}$$

We would like to note here that an apparent horizon depends on the slicing of spacetime. However, we are interested in the issue of global visibility, and at the boundary of collapsing dust ball the apparent horizon coincides with the event horizon. Thus the apparent horizon, which is much easier to locate, should serve our purpose here.

In LTB models, from equations (2.4) and (2.7), we have the following relation between the singularity and the apparent horizon (AH) curve

$$t_{\mathrm{AH}}(r) = t_s(r) - FG(-f). \tag{2.8}$$

This equation is crucial in understanding the causal structure of singularity in LTB models. Since G is a positive and convex function, the positivity of mass F implies $t_{\mathrm{AH}}(r) < t_s(r)$ for the non-central shells ($r > 0$). It is only at the centre that the regularity condition on the initial data demands the mass function F should be zero and hence the singularity and the apparent horizon appear simultaneously, i.e., $t_{\mathrm{AH}}(0) = t_s(0)$. Thus it is only the central singularity which can be a naked shell-focusing singularity. In what follows we shall be interested in the central shell-focusing singularity.

2.2 Initial conditions for stellar collapse

In this paper, we assume the following simple form of the density profile at the initial time $t = 0$,

$$\rho(0,r) = \begin{cases} \rho_c \left(1 - \left(\dfrac{r}{L}\right)^n\right), & 0 \le r \le r_b \\ 0, & r > r_b \end{cases}, \tag{2.9}$$

where ρ_c, L, and r_b ($\le L$) are positive constants, and $n = 1$, 2, or 3. Here, ρ_c is the central density; $r = r_b$ gives the boundary of the star; L is a parameter controlling the gradient of the density as well as n.[1] Since $F(r)$ is twice the Misner-Sharp quasi-local mass, the above ansatz of density distribution fixes $F(r)$ as

$$\frac{1}{2}F(r) = 4\pi \int_0^r \rho(0,r) r^2 dr. \tag{2.10}$$

[1] Since the density profile of neutron stars has a steep cutoff near the surface as is well known [20], this simple choice of the density profile with a discontinuity at the surface is sufficiently good for the present purpose. Moreover, as will be shown later, the trapping of null rays from the singularity by the apparent horizon happens enough inside the star. Thus, we argue that the detail of density profile around the surface does not affect our conclusion.

The LTB spacetime is connected to an outer Schwarzschild spacetime whose mass M is given by

$$M = \frac{1}{2} F(r_b). \tag{2.11}$$

For our purpose, it is convenient to choose (M, ρ_c, r_b, n) as a set of parameters specifying a model. From equations (2.9), (2.10), and (2.11), the remaining parameter L can be given in terms of these parameters

$$L = r_b \left(\frac{12\pi}{(n+3)(4\pi - 3M/\rho_c r_b^3)} \right)^{1/n}. \tag{2.12}$$

Although the set of parameters (M, ρ_c, r_b, n) are almost independent, there are two constraints among them for L to be real and satisfy $L \geq r_b$. From equation (2.12) such constraints are found to be

$$\left(\frac{3M}{4\pi\rho_c} \right)^{1/3} < r_b \leq \left(\frac{n+3}{n} \cdot \frac{3M}{4\pi\rho_c} \right)^{1/3}. \tag{2.13}$$

In the limit that r_b takes the value of left-hand side, L diverges, corresponding to a homogeneous density profile. The equality holds when $L = r_b$, corresponding to that the density vanishes at $r \to r_b-$.

In addition to $F(r)$, the LTB solution has another arbitrary function $f(r)$, which is related to the initial velocity distribution of fluid elements $\dot{R}(0, r)$. We assume that all fluid elements are at rest $\dot{R}(0, r) = 0$ at the initial time, which is called the momentarily static case. From equation (2.2), such a case is realized by taking

$$f = -\frac{F}{r}. \tag{2.14}$$

This condition is very different from the usual "marginally bound" models, where collapsing shells are at rest at spatial infinity, corresponding to vanishing f. The momentarily static condition is astrophysically motivated because the collapse to a black-hole formation will begin when a dynamical instability sets in. Such an instability is due to: the electron-capture at a static massive stellar core; the photo-dissociation reactions at a core of very massive stars in the prompt core-collapse scenario; the cooling and mass fall-back at a newly born neutron star in the delayed collapse scenario [21].

2.3 Null geodesic equation

In order to know the causal structure of spacetimes, it is essential to investigate the behavior of geodesics, especially those emanating from the singularity in the present case. Here, we review the null geodesic equation in the general LTB spacetime (see, e.g., [22] for full descriptions).

We are interested in the null geodesics emanating from the central singularity, which is characterized by $R = r = 0$. In particular, one has to identify the first radial null geodesic emanating from the singularity, which is a part of the Cauchy horizon, out of all possible null geodesics. The central singularity is a singular point of the null geodesic equation from where an infinite family of null rays come out. One needs to parameterize these trajectories appropriately to make each curve have a distinct tangent at the singularity. Such a parametrization is known to be possible in (R, u)-coordinates rather than (t, r)-coordinates, where $u := r^\alpha$ with a constant α (≥ 1) to be fixed for each background so that the null geodesics have finite tangents at the singularity $(R, u) = (0, 0)$.

The equation of trajectory $R = R(u)$ for outgoing radial null geodesics is

$$\frac{dR}{du} = \left(1 - \frac{\sqrt{f + \Lambda/X}}{\sqrt{1+f}}\right) \frac{H(X, u)}{\alpha}, \tag{2.15}$$

where a complete list of variables and functions is

$$\begin{aligned}
u &= r^\alpha, \quad X = \frac{R}{u}, \\
H(X, u) &= (\eta - \beta)X + \left[\Theta - \left(\eta - \frac{3}{2}\beta\right) X^{3/2} G(-PX)\right] \sqrt{P + \frac{1}{X}}, \\
\Theta(r) &= \frac{1 + \beta - \eta}{(1+p)^{1/2} r^{3(\alpha-1)/2}} + \frac{(\eta - 3\beta/2)G(-p)}{r^{3(\alpha-1)/2}}, \\
\eta(r) &= \frac{rF'}{F}, \quad \beta(r) = \frac{rf'}{f}, \quad p(r) = \frac{rf}{F}, \quad P(r) = pr^{\alpha-1}, \quad \Lambda(r) = \frac{F}{r^\alpha}.
\end{aligned} \tag{2.16}$$

It is noted that $\beta(r)$ is defined to be zero when f is zero.

The singularity can be naked if there exists an outgoing null geodesic emanating from it with a definite positive tangent $X_0 := \lim_{u,R\to 0} R/u = \lim_{u,R\to 0} dR/du$. An algebraic equation for such a positive tangent is obtained by taking the limit $(u, R) \to (0, 0)$ in equation (2.15),

$$X_0 = \left(1 - \frac{\sqrt{f_0 + \Lambda_0/X_0}}{\sqrt{1+f_0}}\right) \frac{H(X_0, 0)}{\alpha}, \tag{2.17}$$

where the subscript 0 denotes the limiting value at $r \to 0$. Thus, if there exists a positive root X_0 of equation (2.17) for given initial conditions F and f, the null ray emanating from the singularity with the tangent X_0 is proved to be (a part of) the Cauchy horizon, and therefore, the spacetime is locally naked at least.

3 Astrophysical censorship

As mentioned above, the nakedness of the central singularity is determined by examining the null rays locally around the singularity for given initial conditions F and f. In our models,

Table 1: Input parameters (ρ_c, M, r_b, n) and parameters $(L, X_0, R_{\rm trap}, R_{\rm trap}/R_b)$ obtained from the present analysis. $\alpha = 5/3$ for $n = 1$ and $\alpha = 7/3$ for $n = 2$.

Model	ρ_c [10^{15}g/cm^3]	M [$M_\odot$]	r_b [km]	n	L [km]	X_0 [km$^{1-\alpha}$]	$R_{\rm trap}$ [km]	$R_{\rm trap}/R_b$
A	5.00	1.50	7.00	1	8.98	0.213	0.298	0.0648
				2	7.09	0.0583	0.00170	0.000339
B	2.00	2.00	10.5	1	13.3	0.164	0.533	0.0778
				2	10.6	0.0341	0.00379	0.000502
C	1.50	2.70	12.8	1	16.2	0.144	0.556	0.0656
				2	12.9	0.0263	0.00320	0.000346

introduced in section 2.2, F and f are characterized by the parameters (M, ρ_c, r_b, n). It can be shown that for $n = 1$ and $n = 2$ the singularity is naked, while for $n = 3$ the singularity can be either naked or censored, depending on other factors. Thus, we discuss the $n = 1, 2$ cases and $n = 3$ case separately below. We stress here that the $n = 2$ profile is most physically plausible, since it models the mass distribution of stars in hydrostatic equilibrium. This is because in the presence of pressure $p = p(\rho)$, the pressure gradient can be balanced at the centre with the gravitational force, which is proportional to r, only for $n = 2$. For $n = 1$, the pressure gradient force dominates the gravitational force, while the situation is reversed for $n = 3$. Therefore, we are most interested in the $n = 2$ case, although we also show the result for the $n = 1$ and $n = 3$ cases because the LTB spacetime is a solution describing the collapse of a regular dust cloud even in these cases.

3.1 Weak censorship ($n = 1$ and $n = 2$)

Since we have a wide three-dimensional parameter space of (M, ρ_c, r_b) only constrained by (2.13) even if n is fixed, it is practical to take several sets of numerical values of (M, ρ_c, r_b), each of which is realized by a typical equation of state of neutron star. We adopt three sets of parameters, which we call models A, B, and C (see table 1). Models A, B and C correspond to the marginally stable configurations of neutron stars for an extremely soft, medium and extremely hard equations of state, respectively (see, e.g., [20]).

For a given set of (M, ρ_c, r_b, n), parameter L is determined through equation (2.12). The power α in equation (2.16) is fixed so that $\Theta(r)$ has a finite value in the limit $r \to 0$. The results are $\alpha = 5/3$ for $n = 1$ and $\alpha = 7/3$ for $n = 2$. Solving equation (2.17) with α obtained, a positive root X_0 is determined. Then, one is ready to integrate (2.15) to obtain the trajectory of Cauchy horizon $R = R(u)$ with the initial conditions of $R|_{u=0} = 0$ and $dR/du|_{u=0} = X_0$.

Let us focus on model B with $n = 2$. Qualitative behaviors of the other models are the same as this case. The Cauchy horizon numerically obtained is shown in figure 1. Figures 1(a) and 1(b) show the Cauchy horizon in the (r, R) and (r, t) planes, respectively, with the apparent horizon (2.8). One can see that R increases with r at first along the Cauchy horizon, but then encounters the apparent horizon at $R = R_{\rm trap} = 3.79$ [m] and eventually plunges into the non-central singularity $R = 0$ at $r > 0$. In order to see that this trapping point is enough inside the collapsing star, we define the proper radius of collapsing star at the time of trapping by

$$R_b := R(t_{\rm trap}, r_b), \tag{3.1}$$

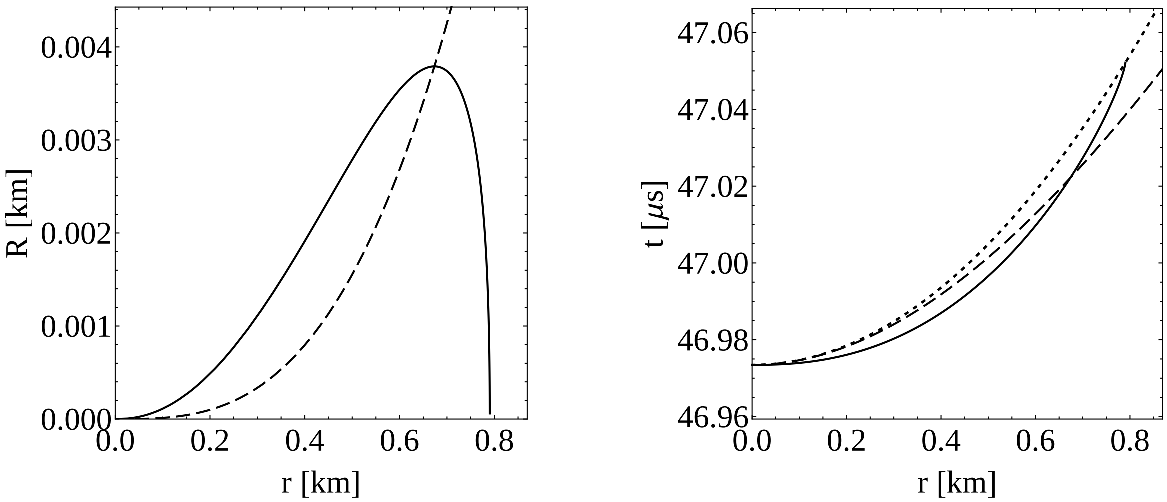
where $t_{\rm trap}$ is the time of trapping measured in the t-coordinate.[2] In the present case, $R_b = 7.56$ [km]. Thus, $R_{\rm trap}/R_b \simeq 0.05\%$ and we can say that the trapping happens enough inside the star. In other words, the naked singularity cannot be globally naked, preserving the weak cosmic censorship hypothesis. A conformal diagram is depicted in figure 2. See table 1 for the values of parameters for the other models.

3.2 Strong censorship ($n = 3$)

In contrast to the cases of $n = 1$ and $n = 2$, where the central singularity is naked necessarily, the nakedness of the singularity for $n = 3$ depends on the other parameters (M, ρ_c, r_b) (it turns out that $\alpha = 3$ in this case). That is, the algebraic equation (2.17) has no positive root for certain parameter regions of (M, ρ_c, r_b), implying that the collapse results in a black hole. As will be shown below, one can see that for any set of (M, ρ_c, r_b) corresponding to an initial condition of astrophysical stellar collapse, algebraic equation (2.17) has no positive root.

In order to reduce the number of parameters and simplify our arguments, let r_b be given by the right-hand side of the inequality (2.13), which results in $L = r_b$. Furthermore, we assume that ρ_c takes the value of model B. Then, the only parameter left to be fixed is the mass M. One can check numerically that the algebraic equation (2.17) has a positive root for $M \leq 0.0688 M_\odot$. Integrating equation (2.15) for a mass below this threshold, one can see also that the Cauchy horizon is never trapped inside the star ($M = 0.0688 M_\odot$ is a marginal case where the Cauchy horizon is trapped just at the surface), implying that the singularity is globally naked. On the other hand, for $M > 0.0688 M_\odot$ equation (2.15) has no positive root, implying the formation of a black hole. Thus, for the collapse of massive stars whose mass is a few solar mass the singularity cannot even be naked, preserving the strong censorship.

[2] Although this proper radius has no coordinate-independent meaning of course, it will do for our present purpose.



(a) (b)

Figure 1: (a) The trapping of the Cauchy horizon (solid) by the apparent horizon (dashed) in the r-R plane for the model B with $n = 2$. The Cauchy horizon emanating from the central singularity $R = r = 0$ is trapped by the apparent horizon at $(r_{\rm trap}, R_{\rm trap}) = (0.673, 0.00379)$, which is enough inside the stellar surface located at $r = r_b = 10.5$, and then plunges into the non-central singularity lying on $R = 0$ at $r = 0.790$. (b) The trapping of the Cauchy horizon (solid) by the apparent horizon (dashed) in the r-t plane again for the model B with $n = 2$. The shell-focusing singularity is drawn by a dotted curve, into which the Cauchy horizon plunges at $(r, t) = (0.790, 47.052)$.

4 Conclusion

We saw in section 3.1 that the naked singularities appearing in the collapse of massive stars cannot be globally naked. Here, we illustrate that globally naked singularities can happen only for parameters that are unrealistic from an astrophysical point of view. As in the argument in section 3.2, let r_b be given by the right-hand side of inequality (2.13), and we take $n = 2$ and the value of ρ_c of model B. Then, only parameter left to be fixed is the mass M. Integrating equation (2.15) numerically up to the surface for various values of M, one can easily find that for $M \geq 0.2466 M_\odot$ the Cauchy horizon is trapped by the apparent horizon before reaching the surface of star as in the cases in section 3.1, whereas for $M < 0.2466 M_\odot$ the Cauchy horizon is not trapped within the stellar interior. Thus, the globally naked singularities can appear only for collapse of the small-mass regime that is far below the masses of the observed neutron stars ($\sim 1.5 M_\odot$) and the maximum mass ($\sim 1.5 M_\odot$) of the neutron star for the extremely soft equation of state.

In this paper, we have modeled the collapse of massive objects by the Lemaître-Tolman-Bondi solution (2.1) with the initial conditions characterized by the initial density profile (2.9),

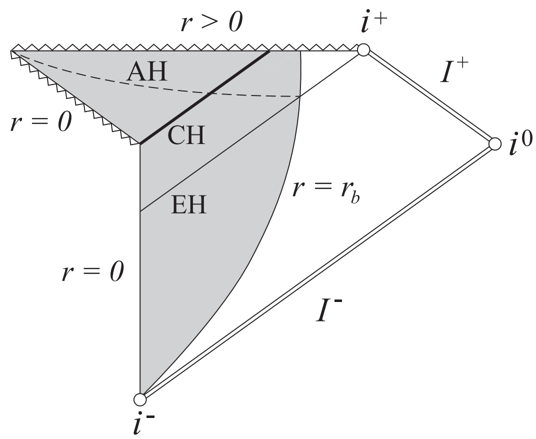


Figure 2: A conformal diagram of the LTB solution mimicking the initial mass distribution of the astrophysically realistic collapse in the final stage of stellar evolution. The central ($r = 0$) singularity is locally naked but not globally. Namely, the Cauchy horizon (CH, thick solid) is trapped by the apparent horizon (AH, thin dashed) and plunges into the non-central ($r > 0$) singularity without reaching the surface of collapsing star ($r = r_b$). The line EH (thin solid) represents the event horizon.

velocity distribution (2.14), and the central density, mass, and radius corresponding to the several models of marginally stable neutron stars (see table 1). For $n = 1$ and $n = 2$ (the latter is the most plausible), the singularity in all models is naked necessarily, but we have shown that it cannot be globally naked. Namely, the first null ray emanating from the singularity cannot reach the future null infinity I^+ (see figure 2), implying that the singularity cannot be seen by any observers except those who plunge inevitably into the future spacelike singularity. Thus, it can be said that the singularity is censored by the astrophysical censor in the weak sense. For $n = 3$, the singularity can be naked or not, depending on the parameters such as mass and central density, but we have shown the singularity cannot even be naked for astrophysically reasonable parameters. Thus, it can be said that the singularity is censored by the astrophysical censor in the strong sense in this case.

Finally, we would like to stress that we are not in the position to claim that there appear locally naked singularities in the astrophysically realistic collapse. The appearance of naked singularities in our analysis is entirely because we choose the dust as a matter model and the situation can be very different in other choice. Instead, our conclusion is that even if a naked singularity could appear at the centre of realistic stellar collapse, it will be hidden behind an event horizon and cannot be observed by a distant observer. Since the exposition of the central naked singularity to a distant observer depends on the dynamics of not central but surrounding region, our conclusion is not very sensitive to the choice of matter models.

Acknowledgments

UM and TH are supported by Research Center for Measurement in Advanced Science in Rikkyo University, and by the Grant-in-Aid for Scientific Research Fund of the Ministry of Education, Culture, Sports, Science and Technology, Japan [Young Scientists (B) 22740176 and 21740190].

References

[1] R. Penrose, "Gravitational collapse and space-time singularities," Phys. Rev. Lett. **14**, 57-59 (1965); S. Hawking, "The occurrence of singularities in cosmology III: Causality and singularities," Proc. Roy. Soc. Lond. **A300**, 187-201 (1967); S. W. Hawking, R. Penrose, "The Singularities of gravitational collapse and cosmology," Proc. Roy. Soc. Lond. **A314**, 529-548 (1970); S. W. Hawking, G. F. R. Ellis, "The Large scale structure of space-time," Cambridge University Press, Cambridge, 1973; J. M. M. Senovilla, "Singularity Theorems and Their Consequences," Gen. Relativ. Grav. **29**, 701-848 (1997).

[2] R. Penrose, Riv. Nuovo. Cimento Soc. Ital. Fis. **1**, 252 (1969); in "Gravitational Radiation and Gravitational Collapse," IAU Symposium **64**, ed. C. DeWitt-Morettee, Reidel, Dordrecht (1974).

[3] L. M. Burko, "Structure of the black hole's Cauchy horizon singularity," Phys. Rev. Lett. **79**, 4958-4961 (1997) [gr-qc/9710112].

[4] M. Dafermos, "The Interior of charged black holes and the problem of uniqueness in general relativity," gr-qc/0307013; M. Dafermos, I. Rodnianski, "A Proof of Price's law for the collapse of a selfgravitating scalar field," Invent. Math. **162**, 381-457 (2005) [gr-qc/0309115].

[5] D. M. Eardley, L. Smarr, "Time function in numerical relativity: Marginally bound dust collapse," Phys. Rev. D **19**, 2239-2259 (1979).

[6] D. Christodoulou, "Violation of cosmic censorship in the gravitational collapse of a dust cloud," Commun. Math. Phys. **93**, 171-195 (1984); R. P. A. C. Newman, "Strengths of naked singularities in Tolman-Bondi space-times," Class. Quant. Grav. **3**, 527-539 (1986); K. Lake, "Smooth (C^2) formation of a persistent naked singularity," Phys. Lett. A **116**, 17-19 (1986); K. Lake, "Naked singularities in gravitational collapse which is not self-similar," Phys. Rev. D **43**, 1416 (1991); P. S. Joshi, I. H. Dwivedi, "Naked Singularities in Non-self-similar Gravitational Collapse of Radiation Shells," Phys. Rev. **D45**, 2147-2150 (1992).

[7] A. Ori, T. Piran, "Naked Singularities In Selfsimilar Spherical Gravitational Collapse," Phys. Rev. Lett. **59**, 2137 (1987); A. Ori, T. Piran, "Self-similar spherical gravitational

collapse and the cosmic censorship hypothesis," Gen. Relat. Gravit. **20**, 7 (1988); A. Ori, T. Piran, "Naked Singularities and Other Features of Selfsimilar General Relativistic Gravitational Collapse," Phys. Rev. **D42**, 1068-1090 (1990); T. Harada, "Final fate of the spherically symmetric collapse of a perfect fluid," Phys. Rev. **D58**, 104015 (1998) [gr-qc/9807038]; T. Harada, H. Maeda, "Convergence to a selfsimilar solution in general relativistic gravitational collapse," Phys. Rev. **D63**, 084022 (2001) [gr-qc/0101064].

[8] P. S. Joshi, "Gravitational collapse: The Story so far", Pramana 55, 529 (2000).

[9] T. Harada, H. Iguchi, K. -i. Nakao, "Physical processes in naked singularity formation," Prog. Theor. Phys. **107**, 449-524 (2002) [gr-qc/0204008]; T. Harada, "Gravitational collapse and naked singularities," Pramana **63**, 741-754 (2004) [gr-qc/0407109].

[10] M. W. Choptuik, "Universality and scaling in gravitational collapse of a massless scalar field," Phys. Rev. Lett. **70**, 9-12 (1993); C. Gundlach, J. M. Martin-Garcia, "Critical phenomena in gravitational collapse," Living Rev. Rel. **10**, 5 (2007) [arXiv:0711.4620 [gr-qc]]; T. Koike, T. Hara, S. Adachi, "Critical behavior in gravitational collapse of radiation fluid: A Renormalization group (linear perturbation) analysis," Phys. Rev. Lett. **74**, 5170-5173 (1995) [gr-qc/9503007]; T. Koike, T. Hara, S. Adachi, "Critical behavior in gravitational collapse of a perfect fluid," Phys. Rev. **D59**, 104008 (1999).

[11] T. Harada, K. -i. Nakao, "Border of spacetime," Phys. Rev. **D70**, 041501 (2004) [gr-qc/0407034].

[12] E. G. Gimon, P. Horava, "Astrophysical violations of the Kerr bound as a possible signature of string theory," Phys. Lett. **B672**, 299-302 (2009) [arXiv:0706.2873 [hep-th]].

[13] K. -i. Nakao, T. Harada, U. Miyamoto, "Visible borders of spacetime generated by high-energy collisions," Phys. Rev. **D82**, 121501 (2010) [arXiv:1007.4610 [gr-qc]].

[14] R. Takahashi, T. Harada, "Observational Testability of Kerr bound in X-ray Spectrum of Black-Hole Candidates," Class. Quant. Grav. **27**, 075003 (2010) [arXiv:1002.0421 [astro-ph.HE]]; C. Bambi, T. Harada, R. Takahashi, N. Yoshida, "Outflows from accreting super-spinars," Phys. Rev. **D81**, 104004 (2010) [arXiv:1003.4821 [gr-qc]]; C. Bambi, K. Freese, T. Harada, R. Takahashi, N. Yoshida, "Accretion process onto super-spinning objects," Phys. Rev. **D80**, 104023 (2009) [arXiv:0910.1634 [gr-qc]].

[15] T. Nakamura, K. Oohara and Y. Kojima, "General Relativistic Collapse to Black Holes and Gravitational Waves from Black Holes," Prog. Theor. Phys. Suppl. **90**, 1 (1987).

[16] B. Giacomazzo, L. Rezzolla, N. Stergioulas, "Collapse of differentially rotating neutron stars and cosmic censorship," Phys. Rev. **D84**, 024022 (2011) [arXiv:1105.0122 [gr-qc]].

[17] S. S. Deshingkar, S. Jhingan, P. S. Joshi, "On the global visibility of singularity in quasi-spherical collapse," Gen. Rel. Grav. **30**, 1477-1499 (1998) [gr-qc/9806055].

[18] G. Lemaître, Ann. Soc. Sci. Bruxelles I A **53**, 51 (1933); R. C. Tolman, "Effect of imhomogeneity on cosmological models," Proc. Nat. Acad. Sci. **20**, 169-176 (1934); H. Bondi, "Spherically symmetrical models in general relativity," Mon. Not. Roy. Astron. Soc. **107**, 410-425 (1947).

[19] J. R. Oppenheimer, H. Snyder, "On Continued gravitational contraction," Phys. Rev. **56**, 455-459 (1939).

[20] S. L. Shapiro and S. A. Teukolsky, "Black holes, white dwarfs, and neutron stars: The physics of compact objects," New York, USA, Wiley (1983) 645 p.

[21] G. E. Brown, H. Bethe, "A Scenario for a large number of low mass black holes in the galaxy," Astrophys. J. **423**, 659 (1994).

[22] P. S. Joshi and I. H. Dwivedi, "Naked singularities in spherically symmetric inhomogeneous Tolman-Bondi dust cloud collapse," Phys. Rev. D **47**, 5357 (1993) [arXiv:gr-qc/9303037]; S. Jhingan, P. S. Joshi, "The Structure of singularity in spherical inhomogeneous dust collapse," gr-qc/9701016.